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Universality, maximum radiation and absorption in high-energy collisions of black holes with spin

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We explore the impact of black hole spins on the dynamics of high-energy black hole collisions. We report results from numerical simulations with γ–factors up to 2.49 and dimensionless spin parameter $\chi = +0.6,\ 0,\ -0.6$. We find that the scattering threshold becomes independent of spin at large center-of-mass energies, confirming previous conjectures that structure does not matter in ultrarelativistic collisions. It has further been argued that in this limit all of the kinetic energy of the system may be radiated by fine tuning the impact parameter to threshold. On the contrary, we find that only about 60% of the kinetic energy is radiated for $\gamma = 2.49$. By monitoring apparent horizons before and after scattering events we show that the "missing energy" is absorbed by the individual BHs in the encounter, and moreover the individual BH spins change significantly. We support this conclusion with perturbative calculations. An extrapolation of our results to the limit $\gamma \to \infty$ suggests that about half of the center-of-mass energy of the system can be emitted in gravitational radiation, while the rest must be converted into rest-mass and spin energy.

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I. Introduction. Historically, one of the primary motivations behind the forty-year long effort to develop numerical relativity was to understand strong-field astrophysical phenomena, such as the mergers of black holes (BHs) and neutron stars (see e.g. [1]). More recently, numerical simulations have begun to shed light on problems of fundamental interest in high-energy physics, such as trans-Planckian scattering and gauge-gravity dualities [2]. A scenario of particular interest in this context is the collision of two BHs near the speed of light. This scenario has been investigated most extensively in $D = 4$ spacetime dimensions for the case of equal-mass, nonspinning BH binaries, which are characterized by two parameters: the boost factor $\gamma = (1-v^2)^{-1/2}$ and the impact parameter $b = L/P$, where v is the center-of-mass velocity, L the initial orbital angular momentum and P the initial linear momentum of a single BH. In the head-on case ($b = 0$) high-energy BH collisions can radiate up to $14 \pm 3\,\%$ of the center-of-mass (CM) energy of the system, and they always produce a nonspinning remnant [3]. Grazing collisions with $b \neq 0$, on the other hand, result in one of the following three outcomes [4]: (i) a prompt merger for sufficiently small $b < b^*$, (ii) a "delayed" merger for $b^* \leq b < b_{\rm scat}$, or (iii) scattering of the holes to infinity for $b \geq b_{\rm scat}$. Here $b_{\rm scat}$ denotes the scattering threshold and $b^* < b_{\rm scat}$ the "threshold of immediate merger": by fine-tuning around b^* a binary can end up in a near-circular orbit for a time $T \propto \log|b - b^*|$, before it either separates or merges to form a single Kerr BH [5]. In [4], grazing collisions with $\gamma \leq 2.9$ were found to radiate as much as $35 \pm 5\,\%$ of the CM energy in gravitational waves (GWs) and result in near-extremal spins without violating the Kerr bound, in agreement with the cosmic censorship conjecture (though this may not be the case in higher dimensions [6]). A parallel study by Shibata et al. [7] investigated the scattering threshold $b_{\rm scat}$ as a function of the CM energy. Their simulations of nonspinning binaries span boosts up to $\gamma = 2.3$ and yield results that are well fitted by $b_{\rm scat} \sim 2.5\,(M/v)$, where $M = \gamma M_0$ is the CM energy (here and below we use geometrical units $G = c = 1$). Comparisons with BH perturbation theory and point-particle collisions in the zero-frequency limit provide a satisfactory understanding of the main qualitative features of these simulations [5, 8, 9], but several outstanding questions remain.

Here we perform a systematic analysis of ~ 120 collisions of spinning and nonspinning BH binaries in four dimensions (see e.g. [6, 10] for early results in higher dimensions) to answer two questions of great relevance for transplanckian physics and BH production from particle collisions, in particular in TeV-scale gravity scenarios [11, 12]: i) is the internal structure of the colliding objects, here consisting of their spin angular momentum, relevant in high-energy collisions? ii) is it possible (as suggested in [5]) to radiate all of the kinetic energy of the system in ultrarelativistic BH encounters?

Our simulations answer both questions in the negative.

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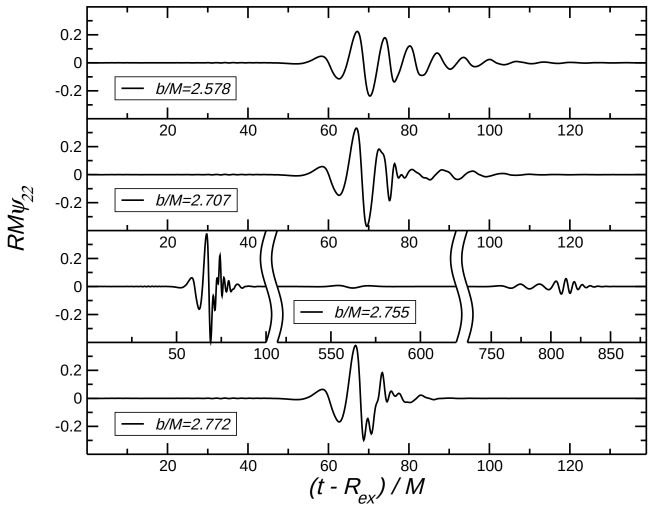

FIG. 1. Four waveforms for $\gamma = 2.49$, antialigned spins and selected values of b. The $b/M = 2.755$ case is a triple encounter (two periastron passages followed by a merger).

We find that spin effects become negligible in the high-energy limit, in the sense that both the scattering threshold and the maximum energy radiated become universal functions of γ (independent of spins). For our largest boost ($\gamma = 2.49$), grazing encounters never radiate more than $\sim 60\%$ of the available kinetic energy of the system; in fact we find this percentage to *decrease* for increasing boost velocity. By monitoring the apparent horizons we show that the "missing" kinetic energy is accounted for by an increase in the BH mass during the encounter. Both observations are important in the context of super-Planckian scattering. They justify the use of classical general relativity to understand properties of the collisions and constrain the amount of GWs radiated which is a prime example of a relevant property, as this will determine the initial mass spectrum of formed BHs.

Our results thus support from a different viewpoint earlier evidence that "matter does not matter" as provided in Ref. [13], where it was shown that the ultrarelativistic collision of two bosonic solitons at sufficiently high energies leads to BH formation. Similar conclusions were reached when colliding self-gravitating fluid objects [14, 15]. These results are consistent with hoop conjecture arguments, which implies that the nonlinear gravitational interaction between the *kinetic energy* of the solitons causes gravitational collapse: the structure of the colliding bodies does not matter.

II. Setup. The simulations presented in this work have been performed with the Lean code [16], which is based on the Cactus computational toolkit [17], uses mesh refinement (provided by Carpet [18]) and the apparent horizon (AH) finder AHFinderDirect [19, 20]. Puncture initial data are provided by a spectral solver [21]. For obtaining stable evolutions of spinning BHs colliding with large boosts, we apply two modifications to the numerical infrastructure employed in previous studies of high-energy BH collisions [3, 4, 22]: (i) following [23], we evolve the conformal factor in terms of the variable $W = \sqrt{\chi}$, and (ii) we reduce the Courant factor to 0.45.

We set up a coordinate system such that the holes start on the x axis with radial momentum P_x and tangential momentum P_y, separated by a distance d. The impact parameter is $b \equiv L/P = P_y d/P$. We extract gravitational radiation by computing the Newman-Penrose scalar Ψ_4 at different radii $r_{\rm ex}$ from the center of the collision. Ψ_4 is decomposed into multipoles ψ_{lm} using spin-weighted spherical harmonics ${}_sY_{lm}$ of spin-weight $s = -2$: $\Psi_4(t, r_{\rm ex}, \theta, \phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} {}_{-2}Y_{lm}(\theta\,, \phi)\, \psi_{lm}(t, r_{\rm ex})$, where θ is measured relative to the x axis.

Estimates of spurious "junk" radiation inherent in the initial data show that it is quite insensitive to the impact parameter, and comparable to our recent findings [3, 4]; we remove it from reported results in a similar manner. Errors due to discretization and finite extraction radius are comparable to those reported in [3, 4]. We estimate uncertainties in radiated quantities of 3 % and 15 % for low and high boosts, respectively. The dominant error contribution arises from finite resolution, and our convergence analysis indicates that our numerical results underestimate the radiated energies.

Contrary to our recent investigation of ultrarelativistic encounters of spinning, equal-mass BHs in "superkick" configurations [22], here we expect the dynamics to be most strongly affected by the "hang-up" effect observed for astrophysical BH mergers with spins (anti)aligned with the orbital angular momentum [24]. In order to investigate the boost dependence of this hang-up effect, we evolve three sequences of equal-mass BH binaries: (i) a sequence with zero spins, (ii) a sequence with spins aligned with the orbital angular momentum and dimensionless spin parameters $\chi = \chi_1 = \chi_2 = 0.6$, and (iii) a sequence with spins of magnitude $\chi = 0.6$ antialigned with the orbital angular momentum. For each sequence we consider four values of the boost parameter ($\gamma = 1.22\,, 1.42\,, 1.88\,, 2.49$) and for each γ we simulate encounters with about 10 different values of b in order to bracket the scattering threshold.

III. Scattering threshold. As mentioned above, we expect a given initial binary configuration to result in either a prompt merger, a delayed merger or scatter to infinity. Our new simulations confirm this scenario. This is illustrated in Fig. 1, where we plot a subset of representative waveforms from the $\gamma = 2.49$ sequence with antialigned spins. For small impact parameter (top) the BHs merge promptly, and the signal is a clean merger/ringdown waveform leading to formation of a BH with dimensionless spin $\chi_f \simeq 0.87$. For the second waveform from the top the merger is not quite prompt: it shows a pattern similar to the scattering waveforms with $b/M = 2.772$ shown in the bottom panel, followed by a ringdown. The third waveform is an example of delayed merger. We chose to display this particular sequence because for $b/M = 2.755$ the binary undergoes a rare *triple* encounter

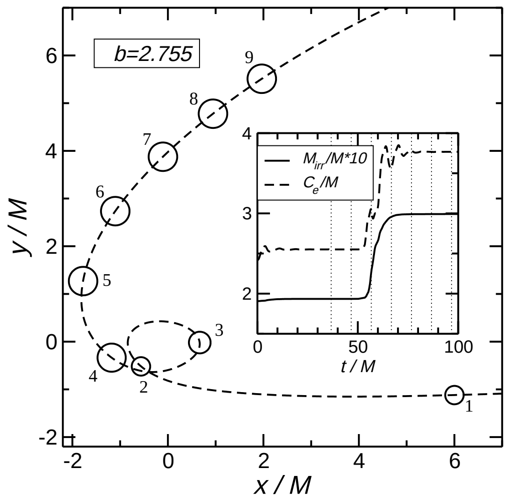

FIG. 2. Trajectory of one BH from antialigned simulations with $b/M = 2.755$ and $\gamma = 2.49$ (cf. Fig. 1). Inset: time evolution of the irreducible mass $M_{\rm irr}$ and of the circumferential radius $C_{\rm e}$ of each hole. The circles represent the BH location at intervals $\Delta t = 10\ M$ (corresponding to vertical lines in the inset) and have radius equal to the irreducible mass $M_{\rm irr}$.

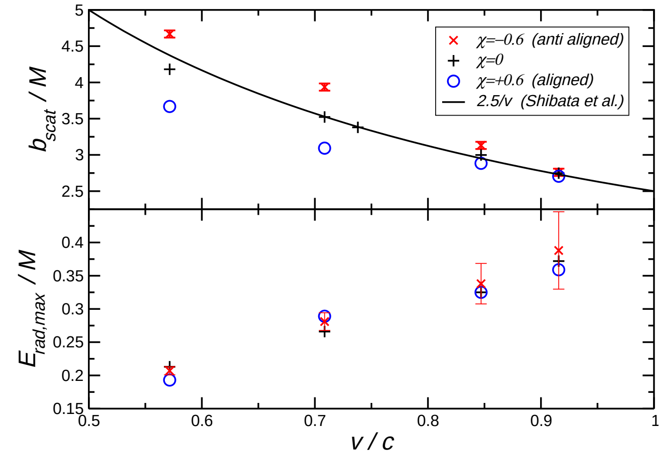

FIG. 3. Critical scattering threshold (upper panel) and maximum radiated energy (lower panel) as a function of γ. Blue circles and red crosses refer to the aligned and antialigned case, respectively. Black "plus" symbols represent the thresholds for the four nonspinning configurations studied in this paper, complemented (in the upper panel) by results from [4] for $\gamma = 1.520$. For clarity, we only plot error bars for the antialigned-spin sequence.

consisting of two revolutions (the second close encounter is visible as a small "bump" at $t/M \sim 550$), followed by a merger signal with relatively low amplitude. Note that the binary radiates and partially absorbs much of the system's kinetic energy during the first scattering encounter, therefore subsequent encounters occur at low velocity and radiate much less. We graphically display this behaviour in Fig. 2, where we plot the trajectory of one BH for the configuration $b/M = 2.755$ and represent snapshots (labeled '1' to '9') of the BH at time intervals $\Delta t = 10\ M$ by circles with radius equal to the irreducible mass. From snapshots '2' to '4' we observe a rapid increase in the "size" of the black hole; successive snapshots are located closer to each other, showing that the BH has slowed down during the encounter.

In order to determine $b_{\rm scat}$ as a function of spin and boost we need to distinguish between merging and scattering collisions. Mergers are easily identified by finding a common AH. We identify an encounter as a scattering case when the following criteria are met: (i) no common AH is found; (ii) the Kretschmann scalar $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ at the origin approaches zero at late times within numerical uncertainties; and (iii) the coordinate trajectories of the BHs, Eq. (14) in Ref. [16], separate out to values comparable to their initial distance.

The scattering thresholds obtained in this way are plotted in the upper panel of Fig. 3. Errors in $b_{\rm scat}$ come from numerical truncation error and discrete sampling of the parameter space, estimated as follows. In the most challenging case ($\gamma = 2.49$) our standard-resolution runs with grid spacing Δx yield $b_{\rm scat}/M = 2.760$ for the antialigned case. By running simulations at two higher resolutions $0.9\ \Delta x$ and $0.8\ \Delta x$, we find $b_{\rm scat}/M = 2.741$ and 2.731 respectively, corresponding to about fourth-order convergence, a Richardson-extrapolated value $b_{\rm scat}/M = 2.713$, and therefore a numerical uncertainty of 0.047. The error due to discretization of the parameter space ($\sim 1.6 \times 10^{-3}$) is negligible in comparison, so we adopt $\delta b_{\rm scat} \approx 0.05$ as a conservative error estimate. Fig. 3 shows that the scattering threshold is spin-independent in the limit $\gamma \to \infty$. Our nonspinning simulations are consistent with the results obtained by Shibata et al. [7] at lower boosts. They are also compatible with the shock-wave analysis in Table II of [25], that suggests a critical impact parameter $b_{\rm scat}/M \gtrsim 1.68$ in the ultrarelativistic limit. Note that the shock-wave construction overestimates the total radiated energy in head-on collisions by a factor ~ 2 [3]; if this is also the case for grazing collisions, one would expect that shock-wave methods may underestimate $b_{\rm scat}/M$ by a similar factor.

IV. Maximum radiation. As pointed out in [4, 26], the total energy $E_{\rm rad}/M$ radiated in grazing BH collisions increases steeply as the impact parameter approaches b^* or $b_{\rm scat}$. This is true also for spinning binaries. For $\gamma = (1.22, 1.42, 1.88, 2.49)$, respectively, we find the maxima in $E_{\rm rad}/M$ plotted in the lower panel of Fig. 3. For reference, the initial fraction of total energy in the form of kinetic energy $K/M = (\gamma - 1)/\gamma$ is $(17.9, 29.4, 46.8, 59.8)\%$, and for the spinning cases we have $(4.2, 3.6, 2.6, 1.7)\%$ in initial spin energy.

For a subset of scattering runs where we monitored the apparent horizon as a function of time, we list estimates of radiated energies and spins before/after scattering in Table I. The accuracy of the radiated energy estimates is limited by discretization errors, the presence of spuri-

ous initial radiation and by our discrete sampling of the impact parameter space, but the numbers clearly reveal two striking features: (i) the maximum radiated energy varies only mildly with spin at any given γ, and (ii) for mild boosts the maximum energy radiated is comparable to the initial kinetic energy of the spacetimes; however as γ increase the ratio drops, down to $\sim 60\%$ for $\gamma = 2.49$. This observation prompts two questions. Where has the remaining kinetic energy gone? Why does the deficit increase with boost?

Spin	γ	b/M	K/M	$E_{\rm rad}/K$	$E_{\rm abs}/K$	$\lvert\chi_{\rm i}\rvert$	$\lvert\chi_{\rm s}\rvert$
$\uparrow$	1.22	4.671	0.179	0.899	0.088	0.60	0.45
$\uparrow$	1.42	3.939	0.294	0.816	0.154	0.59	0.28
$\uparrow$	1.88	3.133	0.468	0.665	0.273	0.59	0.01
$\uparrow$	2.49	2.762	0.598	0.570	0.313	0.57	0.12
0	1.22	4.191	0.179	0.899	0.075	0.01	0.06
0	1.42	3.537	0.294	0.789	0.155	0.02	0.14
0	1.88	3.005	0.468	0.637	0.284	0.08	0.24
0	2.49	2.749	0.598	0.574	0.320	0.10	0.23
$\downarrow$	1.22	3.678	0.179	0.894	0.065	0.60	0.59
$\downarrow$	1.42	3.096	0.294	0.782	0.164	0.59	0.56
$\downarrow$	1.88	2.886	0.468	0.618	0.284	0.59	0.45
$\downarrow$	2.49	2.704	0.598	0.600	0.320	0.57	0.33

TABLE I. Fractional kinetic energy radiated and absorbed for representative numerical simulations with aligned spins ($\uparrow$), antialigned spins ($\downarrow$) or nonrotating BHs ('0'). See the main text for definitions of the various quantities. We also list spin estimates $\chi_{\rm i}$ and $\chi_{\rm s}$ before and after scattering. χ_i is measured at a time $\sim 20\ M$ after the beginning of the simulation, and small deviations from the initial data parameter $j = \pm 0.6$, 0 can presumably be attributed to the BHs absorbing an increasing amount of junk radiation as γ increases.

V. Absorption. The answer to these questions is found in the apparent horizon dynamics of the individual holes before and after the first encounter. We have analyzed the data in detail for a set of scattering events where the individual holes separate sufficiently after first encounter to warrant application of the isolated horizon limit. Specifically, we measure the equatorial circumference $C_{\rm e} = 4\pi M$ and the Christodoulou mass $M_{\rm irr}^2$ of each BH before and after the encounter. The inset of Fig. 2 shows the variation of these quantities with time in a typical simulation: absorption typically occurs over a short timescale $\approx 10M$. Since the apparent horizon area $A_{\rm AH} = 16\pi M_{\rm irr}^2 = [C_{\rm e}^2/(2\pi)](1+\sqrt{1-\chi^2})$, in this way we can estimate the rest mass and spin of each hole before ($M_{\rm i}$, $\chi_{\rm i}$) and after ($M_{\rm s}$, $\chi_{\rm s}$) the first encounter. We define the absorbed energy $E_{\rm abs} = 2(M_s - M_i)$. The results in Table I show that the sum $(E_{\rm rad} + E_{\rm abs})/M$ accounts for most of the total available kinetic energy in the system, and therefore the system is no longer kinetic-energy dominated after the encounter. A fit of the data yields $E_{\rm rad}/K = 0.46(1+1.4/\gamma^2)$ and $E_{\rm abs}/K = 0.55(1-1/\gamma)$, suggesting that radiation and absorption contribute about equally in the ultrarelativistic limit, and therefore that absorption sets an upper bound on the maximum energy that can be radiated.

The fact that absorption and emission are comparable in the ultrarelativistic limit is supported by point-particle calculations in BH perturbation theory. For example, Misner et al. [27] studied the radiation from ultrarelativistic particles in circular orbits near the Schwarzschild light ring, i.e. at $r = 3M(1+\epsilon)$. Using a scalar-field model they found that 50% of the radiation is absorbed and 50% is radiated as $\epsilon \to 0$. We verified by an explicit calculation ignoring self-force effects that the same conclusion applies to *gravitational* perturbations of Schwarzschild BHs (cf. [28]). A recent analysis including self-force effects finds that 42% of the energy should be absorbed by nonrotating BHs as $\epsilon \to 0$ (cf. Fig. 4 of [9]).

Rather than considering particles near the light ring, we can model our problem using particles plunging ultrarelativistically into (for simplicity) a Schwarzschild BH. Davis et al. [29] first computed the energy absorbed when a particle of mass m falls *from rest* into a Schwarzschild BH of mass $M_{\rm BH}$, and they found the remarkable result that the total absorbed energy (summed over all multipoles $\ell \geq 2$) diverges. Physically, the divergence is due to the fact that most of the absorption occurs near the horizon, so we must go beyond the point-particle approximation and introduce a physical cutoff at $\ell_{\rm max} \approx \pi M_{\rm BH}/2m$ to take into account the finite size of the infalling particle. For comparable-mass encounters it is reasonable to truncate the sum at $\ell = 2$. By adapting the BH perturbation theory code of [30], we extended the calculation of [29] to arbitrary spacetime dimensionality and to generic particle energies $p_0 = E/m$. Details of this study will be reported elsewhere. For $D = 4$, and truncating the sum at $l = 2$ as in [29] when we compute absorption, our calculation shows that the radiated (absorbed) energy is $E^{\rm PP}_{\rm rad,abs} = k_{\rm rad,abs}(p_0 m)^2/M_{\rm BH}$, with $k_{\rm rad} = (1.04\times 10^{-2}, 3.52\times 10^{-2}, 0.119, 0.262)$ and $k_{\rm abs} = (0.304, 0.310, 0.384, 0.445)$ for $p_0 = (1, 1.5, 3, 100)$, respectively. So, again, in the UR limit the point-particle model predicts a roughly comparable amount of emission and absorption. As a consequence of this significant energy absorption, in the large-γ limit close scattering encounters between two arbitrarily small (in rest mass) BHs can result in two, slowly moving BHs with rest mass increased by a factor of order γ. Figure 2 provides an explicit example of this scenario, where BHs grow in size due to absorption and slow down after scattering.

Another remarkable implication of Table I is that close high-energy scattering encounters can drastically modify the spin magnitude, presumably due to some combination of absorption and dynamical tidal interactions. For example, when $\gamma = 2.5$ the BH spins decrease from ~ 0.6 to ~ 0.3 in the aligned case, from ~ 0.6 to ~ 0.1 in the antialigned case, and we measure a post-scattering spin $\chi \sim 0.2$ for initially nonspinning encounters. These changes in dimensionless spin parameter correspond to

roughly the *same* total angular momentum being transferred to the black holes during the interaction, independent of the initial spin.

VI. Ultrarelativistic extrapolation and a conjecture. We can now try to estimate the maximum radiated energy when $\gamma \to \infty$ as follows. In Ref. [3], we estimated that for head-on collisions ($b = 0$) we can radiate at most $E_{\rm rad}/M \sim 0.14$ in this limit. For each Lorentz factor γ, the increase in radiation induced by fine-tuning near threshold can be characterized by the ratio $\mathcal{R} \equiv E_{\rm rad}(b = 0)/E_{\rm rad}^{\rm max}$. This ratio increases with boost: for our largest γ it is approximately $\mathcal{R} \sim 0.2$, and a fit to our data yields $\mathcal{R}(\gamma) = 0.34(1 - 1/\gamma)$. Using this fit in combination with results from Ref. [3] we find $E_{\rm rad}^{\rm max}/M \approx 0.14/0.34 \sim 0.41$ as $\gamma \to \infty$. A more conservative upper bound on the total radiated energy at threshold can be obtained assuming that $\mathcal{R}$ is a monotonically increasing function of γ, but using the last data point in our simulations as a lower limit on $\mathcal{R}$: this yields a very conservative upper limit $E_{\rm rad}^{\rm max}/M \lesssim 0.14/0.2 = 0.7$ as $\gamma \to \infty$. These results are consistent (within the errors) with our discussion of Table I, which indicates that radiation in high-energy encounters accounts for roughly 0.46 of the available energy, the rest being absorbed. Therefore our simulations seem to settle the long-standing question of whether it is possible to release all of the CM energy as GWs in high-energy BH collisions: the answer is no.

Two crucial assumptions underlie the study of BH production from high-energy particle collisions. The first assumption, that BHs are indeed produced as a result of the collision, is now on a firmer footing due to the results of Refs. [13–15], where it was verified that the hoop conjecture is valid even in highly dynamical situations. The present study addresses the second crucial assumption, i.e., that the internal structure of the colliding bodies is irrelevant at high energies. Furthermore our simulations provide strong evidence that, because of absorption, the maximum radiation produced in ultrarelativistic encounters in four dimensions cannot exceed $\approx 50\%$ of the CM energy.

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